

# Using spectral analysis and multinomial logit regression to explain households' choice patterns

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**Abstract** Many methods are available to analyze rank-ordered data. We used spectral analysis to identify the most preferred option of Formosan subterranean termites (FSTs) control as ranked by Louisiana homeowners. Respondents were asked to rank four termite control methods from the most preferred option to the least preferred option. Spectral analysis of complete ranked data indicated that the most preferred FST control choice is a relatively cheap (\$0.13/square foot) option of a liquid treatment. Similarly, analysis indicated that liquid and bait treatments are the two most desired control choices. Multinomial logit analysis indicated that survey location, household pre-tax income, and knowledge of FSTs determined Louisiana homeowners' ranking pattern choices.

**Keywords** Complete ranked data · Formosan subterranean termite · Invasive species · Multinomial logit · Rank-ordered data · Spectral analysis

**JEL Classification** C65

## 1 Introduction

Invasive species are non-indigenous species that cause major environmental damage (biodiversity and habitat losses) (Fernandez 2007; Eiswerth and Johnson 2002) and

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are responsible for as much as \$120 billion of damage every year to the U.S. economy (Pimentel et al. 2005). Consider a case of Formosan subterranean termites (FSTs) (*Coptotermes formosanus* Shiraki), a species native to China, which were introduced after the Second World War primarily in the U.S. states of Louisiana, South Carolina, and Texas by returning ships. A single FST colony may consist of 1–10 million termites. They are known to attack structural woods as well as living plants. FST colonies are established not only by their routes from the ground to wooden structures but also through aerial infestation during their swarming season (Su and Scheffrahn 1987). As of 2010, these invasive species have been present in the following U.S. states: Alabama, California (an isolated infestation in San Diego County), Florida, Georgia, Hawaii, Louisiana, Mississippi, North and South Carolina, Tennessee, and Texas. The damage from the FST infestation in the U.S. exceeds \$1 billion per year. In Louisiana alone, the most affected state in the continental U.S., the damage from the FST is about \$500 million a year.

Control of the FST in Louisiana presents a rather unique opportunity at economic analysis, because in our particular case the FST and its control represent analytics concerning a quasi-public good. FST infestation is heavier in the New Orleans area and infestation intensity decreases as we move away from New Orleans. In the historical district area of New Orleans (the 16 block area composing the area known as the French Quarter), the FST control can be thought of as purely of a public good nature, because it is impacting the historical district area and the public sector is sponsoring the control method. Therefore, it makes sense to treat the area with the most effective treatment control method. In fact, this expensive treatment method has been used for FST control in the French Quarter area. Our analysis could support a public choice on incentives to households for FST control in the historical district. This would mean that public authorities have to choose to subsidize a given treatment in an exclusive way and that has been exactly the case. As one moves away to other areas within the city or away from the city to other places in Louisiana the problem has yet to become as serious as it is in the New Orleans area. The treatment choice becomes more like a private good, where individuals can choose whatever the treatment method s/he wants to use in his/her house. Here, the private individuals have exclusionary alternatives. The government's role would be to minimize or prevent infestation in the area which is not already impacted.

Information on complete ranking provides what is the most preferred choice of homeowners as well as the least preferred choice. In our case, not treating the house is definitely the least preferred choice for public officials. Therefore, given a scenario where some kind of treatment is preferred to none at all, where a public entity may be willing to subsidize control efforts to maximize use and minimize further damage, it thus becomes desirable to understand the most and least preferred preferences. In such a case one must look at complete rankings.

We collected data using a contingent ranking method to find preferences for alternative FST control options by Louisiana homeowners. Respondents ranked alternative FST control options in categorical forms with each category reflecting the preference intensity. These types of preference ranking data are often coded as consecutive integers from one to the number of categories to their degree of preference, but the number does not represent their distance. When preference intensity is presented, economists

try to find the factors affecting these rankings but fail to identify the most preferred treatment option. This paper attempts to fill this void in economics literature using a case of FST control options ranked by Louisiana homeowners.

A group of respondents may rank alternative choices as their first, second, third, fourth preference and so on. If respondents rank all items available in a survey, it represents a complete ranking or full ranking. A comprehensive review of complete ranking of data is available in [Diaconis \(1989\)](#) and [Critchlow \(1985\)](#). We used a spectral analysis method to analyze complete ranked preference data. We provide a brief review of how ranked data have been handled in literature. We present a method section detailing the theory of spectral analysis as used in rank-ordered data analysis. We discuss complete details of data features in Sect. 4. We expand the analysis to identify how a complete bundle of rankings are impacted by the demographic and cognitive risk/benefit variables. Supporting materials are relegated in Appendices.

## 2 Relevant literature

There exist several approaches to analyze rank-ordered data [examples are the distance based model of [Critchlow \(1985\)](#) and [Mallows \(1957\)](#); and the multistage ranking model of [Benter \(1994\)](#)]. [Marden \(1995\)](#) and [Greene and Hensher \(2010\)](#) provide details of many rank ordered analysis methods commonly used in literature.

One way of investigating preference ranked data is using a completely randomized factorial design ([Scheirer et al. 1976](#)). This procedure, as an extension of the Kruskal–Wallis rank test, allows for the calculation of interaction effects and linear contrasts. Another way of analyzing ranked data is to use a Markov Chain Monte Carlo technique ([Eriksson 2006](#)). Although these methods are computationally efficient to analyze, those are not easy to conceptualize in different dimensions. [Thompson \(1993\)](#) applies a generalized permutation polytopes and exploratory graphical method for ranked data. The author presents an exploratory graphical method to display frequency distribution for fully ranked data. This method is extremely effective for small number of items, but is ineffective for large  $n$  ([Kidwell et al. 2008](#)). As a result [Kidwell et al. \(2008\)](#) extended permutation polytopes for the visualization of ranking data for large  $n$ , which is easy to use and computationally efficient.

In recent articles, [Gormley and Murphy \(2008, 2010\)](#) developed a mixture model for ranked data which utilizes clusters of homogenous groups to identify the preference ranking pattern. Another recent method developed by [Lee and Yu \(2010\)](#) uses a distance based tree model for ranking data. Their methods can relax the assumption of homogenous population and can incorporate covariates easily. Increased interest in ranked data analysis in recent years is evident from the Neural Information Processing System's learning and ordering workshop. The papers available from this workshop can be found at the Carnegie Mellon University website <http://www.select.cs.cmu.edu/meetings/nips09perm/>.

In economics, the primary method used for ranked data include ordered probit or exploded logit methods ([McFadden 1974](#); [Chapman and Staelin 1982](#); [Paudel et al. 2007](#)). Economic literature has paid less attention to the generalized spectral decomposition method of [Diaconis \(1988, 1989\)](#) despite the fact that it is a very useful method

to analyze fully ranked preference data. Among the few uses of this method in economics and political sciences include papers by [Lawson and Orrison \(2002\)](#) and [Pedrotti et al. \(2006\)](#). [Lawson and Orrison \(2002\)](#) used these ideas to detect hidden coalition in the vote of nine judges of the United States Supreme Court. Recently, [Pedrotti et al. \(2006\)](#) used generalized spectral analysis to find preference for cars. They used first and second order effects to compare preference for different attributes in cars by male and female survey respondents. Like these authors, we also use first and second order spectral analysis to identify the most preferred choice of termite control in Louisiana but supplement the information using an inference based multinomial logit analysis.

### 3 Method

We applied spectral analysis to find the most preferred treatment option for FST control as ranked by homeowners in Louisiana. Spectral analysis is much more like classical two-way ANOVA and ANOVA is a special case of spectral analysis ([Diaconis 1988](#), p. 153). Generalized spectral analysis is used to decompose data on order preference for instance in first and second order effects but ANOVA does not consider order of preference ([Diaconis 1989](#)).<sup>1</sup> Moreover, spectral analysis captures the natural symmetries present in the data that are generally hidden in the existence of a symmetric group. One can interpret the information by decomposing the data according to these symmetries ([Pedrotti et al. 2006](#)). The second order effect detects the positive (or negative) power of combination of two pair attributes ([Pedrotti et al. 2006](#)).

Spectral components will group pair of control options and identify the corresponding pairs to find the preference ([Hannan 1965](#)). Due to these reasons, we must use a spectral decomposition method compared to ANOVA to find most preferred treatment option for FST control. We briefly outline a general theory of spectral analysis applicable for rank-ordered data. Detail treatment of this method can be found on [Serre \(1977\)](#), [Diaconis \(1988, 1989\)](#), and [Iwasaki \(1992\)](#).

Let's assume we have  $n$  types of FST treatment option provided to Louisiana homeowners for ranking denoted by  $i, i = 1, 2, \dots, n$ . Let  $\pi(i)$  denotes the rank given to  $i$ th treatment option. This type of data can be represented using permutation. A permutation  $\pi$  is a bijective function  $\pi : \{1, 2, \dots, n\} \rightarrow \{1, \dots, n\}$  associated with each item  $i \in \{1, \dots, n\}$  and rank  $\pi(i) \in \{1, \dots, n\}$  ([Critchlow 1985](#)). Hence, the number of respondents choosing ranking preference  $\pi$  forms a data set which is denoted by  $f(\pi)$  and can be expressed as

$$f(\pi) = \begin{pmatrix} 1 & 2 & \dots & n \\ \pi(1) & \pi(2) & \dots & \pi(n) \end{pmatrix}.$$

If we are ranking  $n$  items, the permutation of the number of items multiplied by their frequencies gives the sample size of the data for complete ranking. Suppose there are

<sup>1</sup> An astute reviewer has pointed out that one can replicate results obtained from spectral decomposition using proper contrasting, regression, and deriving the residuals under the ANOVA framework. The ANOVA based replication of our results is available from the corresponding author upon request. Papers comparing ANOVA and spectral decomposition are by [Hannan \(1965\)](#), [Jackson and Lawton \(1969\)](#), [Diaconis \(1988\)](#), and [Wu et al. \(2009\)](#).

**Table 1** Preference of FST control options in Louisiana complete rankings

Combinations	Ranking ( $\pi$ )				Number of respondents
	First	Second	Third	Fourth	
1	1	2	3	4	123
2	1	2	4	3	1
3	1	3	2	4	6
4	1	3	4	2	4
5	1	4	2	3	1
6	1	4	3	2	7
7	2	1	3	4	55
8	2	1	4	3	1
9	2	3	1	4	15
10	2	3	4	1	305
11	2	4	3	1	24
12	2	4	1	3	0
13	3	1	2	4	1
14	3	1	4	2	1
15	3	2	1	4	2
16	3	2	4	1	48
17	3	4	1	2	2
18	3	4	2	1	39
19	4	1	2	3	2
20	4	1	3	2	0
21	4	2	1	3	2
22	4	2	3	1	20
23	4	3	1	2	0
24	4	3	2	1	88
Total					747

four available FST treatment options that are provided to respondents to rank from the most preferred to the least preferred. Then, there will be  $4!$  ( $=24$ ) complete rankings. These possible ranking patterns and the number of respondents choosing these ranking patterns are shown in Table 1 for our data. From group theory, we can represent it as a symmetric permutation group denoted by  $S_n$ . Then,  $S_n$  is a finite group operating transitively on  $\pi$ . Let  $V$  be the space of all functions on  $\pi$  with values in real space  $R$ .  $V$  is also a vector space on which  $\pi$  acts linearly as a group transformation  $\sigma f(\pi) = f(\sigma^{-1}\pi)$ . Then,  $k$  subspace of  $V$  are invariant with respect to  $S_n$  for every  $\sigma f(\pi) \in V$  and every  $S_n$  implies that  $f(\pi) \in V$ . Hence,  $V$  decomposes into a direct sum of invariant irreducible subspace, as follows:

$$V = V_0 \oplus V_1 \oplus \cdots \oplus V_k$$

In other words, every function  $f(\pi) \in V$  may be written uniquely as a sum

$$f = f_0 + \cdots + f_k \quad \text{where } f_i \in V_i \text{ and } \sigma f_i \in V_i \text{ for all } \sigma \in V_i.$$

Let  $f(\pi)$  be a set (the number of times  $\pi$  appears in the sample), the spectral analysis is the projection of  $f$  onto the invariant subspaces and the approximation of  $f$  by as many pieces as required to give a reasonable fit (Diaconis 1988). Details on decomposition and computation and first and second order spectral analysis are provided in the Appendix A1.

#### 4 Data

Data were collected by means of a survey of homeowners regarding their preference of FST treatment options in Louisiana. FSTs are an invasive species of termite that is currently present in more than 13 states in the U.S. It has been found that the damage by the species is so severe that infested houses become uninhabitable if not controlled in time. Damage estimates due to FST infestations reach approximately one billion dollars per year (Lax and Osbrink 2003).

Dillman (2000) tailored design method was used to collect survey data. The survey was conducted in 2002. The survey population consisted of all owners occupying homes in four metropolitan areas in Louisiana. These respondents might own single-family houses, multi-family houses, apartments, condominiums, or townhouses. Four metropolitan areas, New Orleans, Baton Rouge, Monroe, and Alexandria, were taken as a stratum of the sample. During the survey period, these cities had 100017, 104149, 38559, and 35386 homeowners, respectively. Selective random samples of 6,000 homeowners were chosen from the sampling frame maintained by Best Mailing List, Incorporated, a private list company. A total of 5,641 single family homeowners were contacted through the use of our mail survey: 1,490 from Monroe, 1,305 from Alexandria, 1,395 from Baton Rouge, and 1,451 from the New Orleans Metropolitan areas. Pre-survey and focus group discussions were conducted before mailing the survey. A survey response rate of 25% was obtained, although not all respondents ranked the treatment options.

Four FST treatment options were provided for each individual homeowner to rank from the most preferred choice to the least preferred choice. The FST treatment choices provided are:

- i. No control option: cost \$0/square foot,
- ii. Liquid treatment option: cost \$0.13/square foot,
- iii. Bait treatment option: \$0.43/square foot,
- iv. Liquid + bait treatment option: \$0.56/square foot.

The details on these treatment choices are given in Appendix A5. Individual homeowners ranked these options as their first, second, third, and fourth most preferred option to control FST. There were a total of 747 observations obtained from the survey in which individuals ranked termite control options completely. The details on the ranking of these options by the respondents are provided in Table 1. The column

entries of Table 1 show the control method ranked in the given permutation. For example, an entry of “1234” means that those respondents ranked the “No Control” option method as their first preferred method, “Liquid Treatment” as their second preferred option, “Bait Treatment” as their third preferred option, and “Liquid+Bait Treatment” as their fourth preferred option.

## 5 Results from spectral analysis

The percentage of respondents ranking preference  $i$  in position  $j$  is shown in Table 2. This table indicates that 52.2% of respondents preferred the liquid treatment option as the first choice and 55.7% of respondent favored Bait as their second choice. The result of first order spectral analysis is shown Table 3 and calculation details are provided in Appendix A2. The largest number 213 in the first column of Table 3 indicates liquid treatment option received the most votes as respondents’ first choice of control option. The largest number in the second column, 231, shows that Bait received the most votes as the second most favorable control option. The liquid + bait treatment option is the third choice and no treatment option is the fourth choice. This means that respondent homeowners want to control FST using some form of control measure. The result of the second order analysis is shown in Table 4 and calculation details are provided in Appendix A3. Each pair of control option can be chosen as six easily interpreted functions. Geometrically, the function projects to 36 points in a four-dimensional space. This means there are only four independent values in the table consisting of 36 values. It is easy to interpret second order unordered effects when there are more choices (greater than four, see Diaconis 1989). Since we have only four-dimensions in second order decomposition, this gives some equal values as shown in Table 4. The largest value 66 in the first column indicates that there is a substantial effect between

**Table 2** Percentage of respondents ranking preference  $i$  in position  $j$

Method	Rank			
	1	2	3	4
No control	22.8	7.98	2.8	70.2
Liquid	52.2	26.5	18.3	1.9
Bait	12.6	55.7	30.7	0.9
Liquid + bait	12.5	9.8	48.2	27

**Table 3** First order effects—complete ranking

Method	Rank			
	1	2	3	4
No control	−45	−127	−166	337
Liquid	213	9	−50	−173
Bait	−94	231	42	−180
Liquid + bait	−75	−114	173	15

**Table 4** Second order, unordered effects

Method	Rank					
	1,2	1,3	1,4	2,3	2,4	3,4
No control and liquid	30	-79	49	49	-79	30
No control and bait	-96	141	-45	-45	141	-96
No control and liquid + bait	66	-62	-4	-4	-62	66
Liquid and bait	66	-62	-4	-4	-62	66
Liquid and liquid + bait	-96	141	-45	-45	141	-96
Bait and liquid + bait	30	-79	49	49	-79	30

control methods Liquid and Bait in ranking (1, 2). For pairs of methods like Liquid and Liquid + Bait, there is an opposite effect: every homeowner likes both or dislikes both treatment options because the row entry begins and ends (-, -) with the same value. Based on the highest value of liquid treatment option and bait treatment option in the second ordered effect and the highest value of liquid treatment option in first ordered effect, it can be said that these two are the two most desirable treatment options chosen by homeowners in Louisiana.

The foregoing analysis shows that homeowners prefer treatment 2 (liquid treatment option) based on the first order analysis. It also tells us that the two most preferred choices are liquid treatment and bait treatment. However, it does not tell us how different respondent characteristics affect the choice pattern. Further analysis indicated that both respondent groups (YES to socio-economic characteristics and NO to the socio-economic characteristics) prefer treatment 2 (liquid treatment) although the strength of preference is low as we go from YES to NO categorical characteristics. For example, an individual living in New Orleans ranks treatment 2 (liquid treatment) as the most preferred option, which is the same case for homeowners living outside of New Orleans; however, the strength of preference value is less in the latter group. Using second order analysis, we found the (liquid treatment (2), bait treatment (3)) as the two most preferred options. However, these choice patterns did not vary by socio-economic characteristics.<sup>2</sup> This has created a need to look into the preference issue even further, which we expand by using a Mallow's approach and a multinomial logit regression.

### 5.1 Mallow's approach

We can be more precise on overall preference of the FST treatment option given what respondents preferred in the first and second order analyses. We used the Mallow's method to further evaluate this contribution. The result of preference for different combinations is given in Table 5. An illustration for income category is given in Appendix A4.

<sup>2</sup> Tables presenting these results are not shown here but are available from the corresponding author.



**Table 5** Preference of a single treatment method

Variables	FSTs control options	Category	
		0	1
Annual pretax household income: \$125K or more (1 = yes)	1	-129.5	-42
	2	<b>206.5</b>	16
	3	107.5	<b>30</b>
	4	-184.5	-4
Survey location New Orleans (1 = yes)	1	-106.5	-65
	2	<b>186.5</b>	36
	3	89.5	<b>48</b>
	4	-169.5	-19
Concrete slab home foundation (1 = yes)	1	-44	-127.5
	2	<b>48</b>	<b>174.5</b>
	3	42	95.5
	4	-46	-142.5
Home market value \$300K or more (1 = yes)	1	-133.5	-38
	2	<b>211.5</b>	11
	3	104.5	<b>33</b>
	4	-182.5	-6
Termites an existing problem in neighborhood (1 = yes)	1	-51	-120.5
	2	<b>143</b>	79.5
	3	37	<b>100.5</b>
	4	-129	-59.5
Gender female (1 = yes)	1	-104	-61.5
	2	<b>143</b>	<b>73.5</b>
	3	76	56.5
	4	-115	-68.5
Heard of FST (1 = yes)	1	-6	-164.5
	2	<b>47</b>	<b>173.5</b>
	3	11	127.5
	4	-52	-136.5

Bold values in column titled "Category" indicates preferred treatment option

Table 5 shows that a treatment option choice differs by socio-economic characteristics. Consider the case of preference difference by an income category (variable name: annual pretax household income). The liquid and bait options are positive for both income categories. The effects of both liquid and bait treatments are greater in the respondent category having household income below \$125K. The absolute values of all treatment options are higher in this income category as well. The highest value of 206.5 in the liquid treatment option shows that respondents with income less than \$125K prefer the liquid treatment option, whereas the respondents with more than \$125K income prefer the bait treatment option. We present these values in Table 5 and provide the calculation details in the Appendix A4. We found that if home market

value is less than \$300K, respondents prefer the liquid treatment option, but respondents prefer bait as a treatment option if their household market value is greater than or equal to \$300K. In addition, households that consider termites to be an existing problem in their neighborhoods prefer the bait treatment option, whereas the liquid treatment option is preferred by those who do not consider termites to be an existing problem in their neighborhoods. Finally, respondents from New Orleans preferred the bait treatment option whereas respondents from outside New Orleans chose liquid as a treatment option. For other socio-economic-physical respondent characteristics, we found no difference between two categories studied as reflected from the results presented in Table 5.

## 6 Multinomial logit regression

So far we have presented results based on the data-analytic method.<sup>3</sup> One must also consider inferential aspects which depend on a probabilistic model. One of the most frequently observed inferential based methods in the existing economics literature for ranked data is to analyze only the first choice using an ordered probit model. When an individual ranks alternatives in order such as ranking the most preferred first, the second most preferred second and so on, until all  $n$  choices are ranked, the most frequently observed model is the rank-ordered logit model developed by [Beggs et al. \(1981\)](#). If we have both characteristics of choice and characteristics of respondents, we can use an exploded logit model or rank-order logit model. If there are specific characteristics associated with choice, then an attribute-specific random order probit or random order logit model can be used. In our case, the difference between the four treatment methods revolve around the pest control operator's monitoring frequency and the total cost of each treatment option. Some (e.g., [Johnston and Roheim 2006](#)) have interacted these choice specific variables with socio-demographic variables and identified the characteristics affecting the ranking pattern. We argue that it may not be correct to employ this type of model to identify the variable affecting ranking patterns, because a priori we lack information regarding whether or not these variables interact. Therefore, we have chosen a different analytical approach for analyzing choice data.

As indicated in the earlier section, there are  $4!$  (or 24) possible orderings of four FST treatment methods by an individual household. Out of these 24 individual rankings, Louisiana homeowner survey respondents chose only 20 different ranking patterns. It is also evident from Table 1 that only a few ranking combinations were chosen by a large majority of respondents. Based on these responses, we can develop eight distinct choice patterns and one "fringe choice pattern" consisting of all remaining choice patterns. We combined these fringe choice patterns into one group called "other." Therefore, we have a total of nine choice combinations. We identify variables that affect a respondent's distinct ranking combinations based on their socio-economic characteristics.

The basic analytical framework was provided by the random utility model. Let  $U_{ij}$  denote homeowner  $i$ 's utility from choosing alternative  $j$  ranking pattern. Then,

<sup>3</sup> [Gormley and Murphy \(2010\)](#) have used clustering and regression to analyze ranked data.

the homeowner  $i$  chooses alternative  $j$  if  $U_{ij} > U_{ik}$  for all  $k \neq j$ . It is standard to assume that  $U_{ij} = V_{ij} + \varepsilon_{ij}$  where  $V_{ij}$  is the deterministic components of the utility and  $\varepsilon_{ij}$  is the random component that represents the researcher's ignorance about the consumer utility function. Assuming  $\varepsilon$ s are independent and have a type I extreme value distribution, the model for the ranking bundle is

$$\Pr(Y_i = j) = \frac{e^{V_{ij}}}{\sum_{k=1}^9 e^{V_{ik}}}, \quad k = 1, \dots, 9.$$

Here, the respondent  $i$ 's observed choice ( $Y_i$ ) takes the value 1 through 9 depending on how he/she ranks the different treatment option in the ordering. The log likelihood function for the multinomial logit model was then given by

$$\ln L = \sum_{i=1}^n \sum_{j=1}^9 d_{ij} \ln \frac{e^{V_{ij}}}{\sum_{k=1}^J e^{V_{ik}}}$$

The empirical model was obtained by specifying the component in the vector  $x_i$  of  $V_{ij} = x_i \beta_j$ . We included a constant, LOCATION, MKTVAL, HOMFOUND, TERMNEIGH, FSTHEARD, GENDER, AGE, EDU, INCOME, ETHNIC. These are both demographic variables and variables that measure a respondent's risk and benefit perceptions regarding termite infestation. Generally speaking, we expect that the market value of a house (MKTVAL) will have a positive sign, because we hypothesize that residents owning more expensive homes are more likely to pay for termite control. We assume that homeowners owning homes with concrete slab foundations (HOMFOUND) are less likely to pay for termite control because homes with slab foundations are pre-treated at construction and because there may be a perception that concrete slab foundations are "safer" and "more protected" against termite infestation. Therefore, we expect the sign to be negative. We also hypothesize that those homeowners responding to our survey and stating that they consider termites to be a problem in their neighborhoods (TERMNEIGH) are more likely to pay for termite control. Therefore, we expect the sign to be positive. Education and income are hypothesized to have a positive impact on willingness to pay.

## 7 Results from multinomial logit

An important feature of the multinomial logistic regression coefficient is that it estimates  $k - 1$  models, where  $k$  is the number of levels of the dependent variable (in our case  $k = 9$ ). Since the parameter estimates are relative to the reference group, the standard interpretation of the multinomial logit is that for a unit change in the explanatory variable, the logit of the outcome  $m$  relative to the reference group is expected to change by its respective parameter estimate given the variables in the model are held constant. Since most of the variables we have included in the multinomial regression model are binary in nature, we will describe the results obtained from the relative risk

ratio (RR) (or odds ratio). Odds ratios are obtained by exponentiating the multinomial logit coefficients.

We used a ranking ordered  $1 > 2 > 3 > 4$  as the base category as this is the ranking pattern provided by people who preferred no treatment option as their first choice. Their least preferred choice was to pay \$0.56/square foot per year for the most costly FST treatment option as evidenced by their ranking of it as the last choice.

Before analyzing the data, we tested for the independence of irrelevant alternatives (IIA), an assumption that states by removing any categories from the choice set the probability of ranking the remaining categories stays unchanged. According to Hausman test statistics, we found that we could not reject the null hypothesis; hence, the IIA assumption holds.

We present the results from the multinomial logit analysis in Table 6. This table shows coefficients from the regression model, relative risk ratios (odds ratio), and marginal effects. The values in the parentheses below the coefficients are  $p$  values. Coefficient significance holds in most cases between the multinomial logit coefficient and relative RR coefficients, although the same is not true for marginal effects.<sup>4</sup> Coefficients associated with survey location New Orleans are positive and significant in six of the seven choice categories. The range of RR value is between 2.391 and 7.345. This is the relative RR comparing preference of residents in New Orleans to residents of other locations for each choice category relative to the base category, given that the other variables in the model are held constant. For residents in New Orleans compared to other locations in the state, the relative risk for those choosing the  $4 > 3 > 2 > 1$  category relative to base category would be expected to increase by a factor of 7.345 given the other variables in the model are held constant.

Whether or not respondents' homes are constructed with a concrete slab foundation is significant in only one case—the case where respondents with a concrete slab foundation are likely to choose  $2 > 4 > 3 > 1$  by a factor of 3.613 relative to the base category given the other variables in the model are held constant. Although owning a home with a concrete slab foundation should theoretically reduce the risk of termite infestation, respondents were still likely to choose this option (an option that incurs costs) compared to the base option (an option that does not incur costs). Respondents owning a house with a market value over \$300,000 were likely to choose ranking bundle  $2 > 3 > 4 > 1$  by a factor of 6.2412 and  $2 > 1 > 3 > 4$  by a factor of 2.915 compared to those respondents who preferred the base category given the other variables in the model are held constant. Those who indicated that termites were an existing problem in their neighborhoods were most likely to choose  $2 > 3 > 4 > 1$  (by a factor of 3.848 times) compared to those choosing the base category. This coefficient was positive and significant in six out of the seven ranking bundles. Those respondents who had heard of the FSTs were likely to choose a ranking pattern  $4 > 3 > 2 > 1$  by a factor of 7.013 than those who chose the base category. We also found that these individuals were less likely to choose the “other” category. Females were likely to choose ranking bundle  $3 > 4 > 2 > 1$  by a factor of 1.488 and  $4 > 2 > 3 > 1$  by a

<sup>4</sup> Powers and Xie (2000) recommend using the odd-ratios for interpretation since the marginals may not have the same sign as the coefficients. We obtained marginals using STATA's `margins` command with `dydx(.)` option.

Table 6 Results of multinomial logistic regression, FSTs control preferences (base category: 1 &gt; 2 &gt; 3 &gt; 4)

Variables	4 > 3 > 2 > 1			2 > 3 > 4 > 1			2 > 1 > 3 > 4			2 > 4 > 3 > 1		
	Coef.	RR	Marginal effects	Coef.	RR	Marginal effects	Coef.	RR	Marginal effects	Coef.	RR	Marginal effects
N			88			305			55			24
Intercept	-3.931** (0.017)			-4.352** (0.000)			-1.398 (0.140)			-3.123** (0.043)		
Survey location New Orleans (1 = yes)	1.994** (0.002)	7.345** (0.002)	0.025 (0.127)	-0.311 (0.596)	0.732 (0.596)	-0.034** (0.004)	0.959* (0.067)	2.608* (0.067)	0.010 (0.717)	1.816** (0.004)	6.150** (0.004)	0.028 (0.146)
Concrete slab home foundation (1 = yes)	0.826 (0.204)	2.285 (0.204)	0.009 (0.194)	-0.589 (0.190)	0.555 (0.190)	-0.033 (0.112)	0.309 (0.478)	1.362 (0.478)	0.013 (0.555)	1.284* (0.070)	3.613* (0.070)	0.020** (0.035)
Home market value \$300K or more (1 = yes)	-1.633 (0.169)	0.195 (0.169)	-0.016** (0.020)	1.831** (0.003)	6.242** (0.003)	0.128** (0.016)	1.070* (0.081)	2.915* (0.081)	0.083* (0.098)	0.003 (0.997)	1.003 (0.997)	-0.004 (0.759)
Termites an existing problem in neighborhood (1 = yes)	0.993* (0.085)	2.699* (0.085)	0.007 (0.391)	1.347** (0.002)	3.848** (0.002)	0.033** (0.028)	0.724* (0.058)	2.062* (0.058)	0.013 (0.517)	-0.071 (0.890)	0.931 (0.890)	-0.012 (0.210)
Heard of Formosan subterranean termites (1 = yes)	1.948* (0.072)	7.013* (0.072)	0.017** (0.027)	0.812 (0.188)	2.252 (0.188)	0.017 (0.295)	0.517 (0.276)	1.677 (0.276)	0.013 (0.577)	1.216 (0.132)	3.374 (0.132)	0.015 (0.131)
Gender female (1 = yes)	0.479 (0.379)	1.614 (0.379)	0.002 (0.756)	0.480 (0.251)	1.616 (0.251)	0.006 (0.660)	0.022 (0.955)	1.022 (0.955)	-0.019 (0.322)	-0.814 (0.179)	0.443 (0.179)	-0.021** (0.029)
Age of respondents in years	0.005 (0.833)	1.005 (0.833)	0.000 (0.885)	0.037** (0.020)	1.038** (0.020)	0.001** (0.027)	-0.010 (0.510)	0.990 (0.510)	-0.001 (0.150)	-0.017 (0.428)	0.983 (0.428)	-0.001 (0.216)
Education: some college or more (1 = yes)	-1.232** (0.048)	0.292** (0.048)	-0.026 (0.157)	0.654 (0.342)	1.923 (0.342)	0.023 (0.134)	0.130 (0.796)	1.139 (0.796)	0.014 (0.573)	0.758 (0.359)	2.133 (0.359)	0.014 (0.163)
Annual pretax household income: \$125K or more (1 = yes)	2.546** (0.002)	12.751** (0.002)	0.044 (0.151)	0.684 (0.309)	1.982 (0.309)	-0.010 (0.501)	0.171 (0.810)	1.187 (0.810)	-0.041** (0.043)	2.066** (0.004)	7.894** (0.004)	0.034 (0.171)
Caucasian ethnic group (1 = yes)	-1.315** (0.021)	0.268** (0.021)	-0.028* (0.088)	-0.984* (0.052)	0.374* (0.052)	-0.047* (0.095)	-0.538 (0.219)	0.584 (0.219)	-0.035 (0.240)	-1.174** (0.033)	0.309** (0.033)	-0.033* (0.095)

Table 6 continued

Variables	3 > 2 > 4 > 1			3 > 4 > 2 > 1			4 > 2 > 3 > 1			Other categories		
	Coeff.	RR	Marginal effects	Coeff.	RR	Marginal effects	Coeff.	RR	Marginal effects	Coeff.	RR	Marginal effects
N			48			39			20			45
Intercept	-0.406 (0.625)			-0.672 (0.244)			-3.599** (0.000)			-1.080 (0.215)		
Survey location New Orleans (1 = yes)	0.238 (0.706)	1.269 (0.706)	-0.028 (0.193)	0.872** (0.025)	2.391** (0.025)	0.025 (0.639)	1.387** (0.002)	4.001** (0.002)	0.071** (0.049)	0.968* (0.086)	2.633* (0.086)	0.009 (0.737)
Concrete slab home foundation (1 = yes)	-0.089 (0.821)	0.915 (0.821)	-0.012 (0.562)	0.164 (0.545)	1.178 (0.545)	0.026 (0.575)	0.122 (0.733)	1.130 (0.733)	0.001 (0.957)	-0.010 (0.981)	0.990 (0.981)	-0.007 (0.738)
Home market value \$300K or more (1 = yes)	-0.223 (0.800)	0.800 (0.800)	-0.020 (0.515)	-0.021 (0.966)	0.980 (0.966)	-0.093 (0.168)	0.141 (0.800)	1.152 (0.800)	-0.006 (0.866)	-0.958 (0.295)	0.384 (0.295)	-0.045** (0.024)
Terminates an existing problem in neighborhood (1 = yes)	-0.564 (0.172)	0.569 (0.172)	-0.062** (0.002)	0.580** (0.018)	1.785** (0.018)	0.026 (0.528)	1.156** (0.000)	3.178** (0.000)	0.069** (0.008)	0.666* (0.092)	1.947* (0.092)	0.008 (0.665)
Heard of Formosan subterranean termites (1 = yes)	0.141 (0.717)	1.151 (0.717)	-0.010 (0.632)	0.499* (0.070)	1.647* (0.070)	0.086* (0.093)	0.497 (0.238)	1.644 (0.238)	0.019 (0.539)	-0.910** (0.025)	0.402** (0.025)	-0.105** (0.004)
Gender female (1 = yes)	0.194 (0.580)	1.214 (0.580)	-0.007 (0.656)	0.397* (0.095)	1.488* (0.095)	0.036 (0.366)	0.769** (0.015)	2.158** (0.015)	0.052* (0.050)	0.320 (0.395)	1.378 (0.395)	0.000 (1.000)
Age of respondents in years	-0.012 (0.406)	0.988 (0.406)	-0.001 (0.101)	0.008 (0.389)	1.008 (0.389)	0.000 (0.954)	0.034** (0.007)	1.034** (0.007)	0.003** (0.002)	0.009 (0.563)	1.009 (0.563)	0.000 (0.937)
Education: some college or more (1 = yes)	0.182 (0.676)	1.200 (0.676)	0.014 (0.418)	-0.124 (0.674)	0.883 (0.674)	-0.015 (0.781)	-0.264 (0.529)	0.768 (0.529)	-0.020 (0.601)	-0.390 (0.376)	0.677 (0.376)	-0.019 (0.456)
Annual pretax household income: \$125K or more (1 = yes)	-0.905 (0.429)	0.405 (0.429)	-0.061** (0.000)	0.957* (0.057)	2.603* (0.057)	-0.012 (0.855)	1.928** (0.001)	6.877** (0.001)	0.146** (0.010)	1.156 (0.122)	3.176 (0.122)	0.010 (0.774)
Caucasian ethnic group (1 = yes)	0.080 (0.854)	1.084 (0.854)	0.009 (0.637)	0.238 (0.430)	1.268 (0.430)	0.137** (0.005)	-0.256 (0.516)	0.774 (0.516)	-0.019 (0.555)	-0.024 (0.957)	0.976 (0.957)	0.003 (0.869)

\*, \*\* and \*\*\* represent significant at the  $\alpha = 0.1, 0.05,$  and  $0.01$  levels, respectively



factor of 2.158 compared to the base category given the other variables in the model are held constant. We hypothesize that this indicates that females are more likely to choose what they perceive to be a “better” termite treatment package because they are more risk averse in the context of this study when compared to males in the study. Older respondents were likely to choose  $2 > 3 > 4 > 1$  ranking compared to the base category. The odds ratio of choosing this ranking increased by a factor of 0.037 for every one year increase in respondent’s age given the other variables in the model are held constant. College education was insignificant in most of the cases and negatively significant in one case. These individuals were less likely to choose ranking pattern  $4 > 3 > 2 > 1$ . Perhaps this indicates that more highly educated respondents did not perceive that potential marginal benefits incurred from more costly termite treatments outweighed the additional costs or that these more educated respondents were less risk averse. Respondents with higher incomes were likely to choose ranking pattern  $4 > 3 > 2 > 1$  by a factor of 12.751 than respondents choosing the base category given the other variables in the model are held constant. Caucasians were less likely to choose the “other” ranking pattern than the base category.

## 8 Conclusions

The contributions of this study are twofold. First, a generalized spectral analysis method was applied to identify the preference of Louisiana homeowners for four FST control options. The first and second order analyses showed that the liquid treatment option was the most preferred option to control FSTs and the liquid and bait treatment options are the two most preferred options. Second, we took an unusual but comprehensive approach to identifying which factors affect termite treatment ranking patterns in treatment bundle options. To identify that, we estimated a multinomial logit regression model. Our results indicated that most of the demographic groups preferred a choice ranking other than the base “no cost first” category. The highest odds ratio coefficients were contributed by variables such as whether or not a survey respondent was from New Orleans, whether or not a respondent had heard of the FST, and whether or not a respondent had a pretax income greater than \$125,000. Awareness of the government—subsidized FST control program in the New Orleans French Quarter increased the likelihood of choosing a higher cost treatment option.

If the federal government is to continue subsidizing termite treatments, this study indicates that their subsidy efforts should be concentrated on making liquid barrier treatments most available compared to other treatment alternatives, because this is the option that will most likely be chosen by potential participants and chances for success will be greatest.

This study revealed that New Orleans respondents preferred expensive termite treatment options. This could be due to several factors, including a “subsidy effect” that occurs because some areas in that city are already under subsidized termite control. It could also be due to an “information effect” resulting from heavy damages that have occurred in New Orleans over the past 20 years. Regardless, the result is that, from a policy perspective, in order for a subsidy to have a desired effect of increasing control, a greater subsidy would have to be paid to homeowners in New Orleans

than in other cities. Perhaps a tiered subsidy system in which New Orleans residents receive a higher subsidy and other residents in the state receive a smaller subsidy may help prevent further termite infestation. Alternately, rather than some sort of cash or in-kind subsidy, the subsidy provided could be in the form of information, knowledge, and education. Using the information alternative, citizens in Louisiana could be educated regarding the need for treatment and the currently existing treatment options. This study reveals that the information imparted should be targeted to different groups in Louisiana according to where they live, their prior experience with termites, and other demographic categories that relate to termite control option preferences and risk tolerances. The primary conclusion is that we were able to successfully employ new analytical techniques that allow us to make these more specific policy recommendations with a greater degree of confidence regarding their potential for success.

### Appendices

#### Appendix A1: More about spectral decomposition

Let  $V$  be the space of all real valued function on the symmetric group  $S_4$ . Young’s rule is used to determine irreducible subspace in the spectral decomposition as shown in Table A1. So, the space  $V$  decomposes uniquely into the direct sum of five subspaces. Following Diaconis (1989) notation we can express the five subspaces with their dimension in the following table.

$V$	$=$	$V_0$	$\oplus$	$V_1$	$\oplus$	$V_2$	$\oplus$	$V_3$	$\oplus$	$V_4$
$M^{1111}$		$S^4$		$3S^{3,1}$		$2S^{2,2}$		$3S^{2,1,1}$		$S^{1,1,1,1}$
Dim 24		1		9		4		9		1

**Table A1** Young Tableaux for subspaces

Sub Space			Young Tableaux notation																		
Constant	[4]	V1	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>3</td><td>2</td><td>1</td></tr></table>	4	3	2	1														
4	3	2	1																		
First Order	[3,1]	V2	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>2</td><td>1</td></tr><tr><td>3</td><td></td><td></td></tr></table> <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>3</td><td>1</td></tr><tr><td>2</td><td></td><td></td></tr></table> <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>3</td><td>2</td></tr><tr><td>1</td><td></td><td></td></tr></table>	4	2	1	3			4	3	1	2			4	3	2	1		
4	2	1																			
3																					
4	3	1																			
2																					
4	3	2																			
1																					
Second Order (Unordered)	[2,2]	V3	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>3</td></tr><tr><td>2</td><td>1</td></tr></table> <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>2</td></tr><tr><td>3</td><td>1</td></tr></table>	4	3	2	1	4	2	3	1										
4	3																				
2	1																				
4	2																				
3	1																				
Second Order (Ordered)	[2,1,1]	V4	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>3</td></tr><tr><td>2</td><td></td></tr><tr><td>1</td><td></td></tr></table> <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>1</td></tr><tr><td>3</td><td></td></tr><tr><td>2</td><td></td></tr></table> <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>2</td></tr><tr><td>3</td><td></td></tr><tr><td>1</td><td></td></tr></table>	4	3	2		1		4	1	3		2		4	2	3		1	
4	3																				
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Sgn $\pi$	[1,1,1,1]	V5	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td></tr><tr><td>3</td></tr><tr><td>2</td></tr><tr><td>1</td></tr></table>	4	3	2	1														
4																					
3																					
2																					
1																					



$V_0$  is the set of constant functions with one dimension. Second,  $V_1$  is first order effect<sup>5</sup> with nine dimensional space and orthogonal to  $V_0$ . Similarly,  $V_2$  is the second order unordered effect<sup>6</sup> with four dimensional space and orthogonal to  $V_0 \oplus V_1$ .  $V_3$  is second order ordered effect with nine dimensional space and orthogonal to  $V_3$ . The final space  $V_4$  is spanned by the function sign  $\pi$  which is  $\pm 1$  as  $\pi$  can be written with an even or odd number of transpositions (Diaconis 1989).

As mentioned previously, the spectral analysis is the projection of  $f$  onto the invariant subspaces. This type of projection is also called isotypic projection. Many researchers use spectral analysis in time series data where the dimensions are smaller and easy for computation. However, rank-ordered data have higher dimensions compared to time series data, so we cannot find the orthogonal basis to compute projection in the isotypic subspace easily. Mallows (1957) provides an approach to deal with such difficulty. This paper uses his approach to compute both first and second order analyses. We use inner products to compute the final projection of the data

$$\langle f_1 | f_2 \rangle = \sum_{\pi} f_1(\pi) f_2(\pi).$$

### First order analysis

The space  $V_0$  is the set of constant function that is the average frequency of the data, so it has one dimension. The space  $V_2$  is the space of first order function evaluated using Mallows' approach. Therefore, consider a function

$$\pi \rightarrow \delta_{i\pi(j)} \begin{cases} =1 & \text{if } \pi(j) = i \\ =0 & \text{otherwise} \end{cases}$$

where  $i$  is the control method and  $j$  is the rank given to the control method.

The first order function becomes  $\sum_{i,j} a_{ij} \delta_{i\pi(j)}$ . In order to get direct sum decomposition, the coefficients should satisfy the following condition

$$\sum_{i,j} a_{ij} = 0.$$

If we consider our data set, it consist of three three-dimensional subspaces, so it projects a nine-dimensional space which can be shown using hook-length formulae following Young's rule as presented in Table A1.

### Second order analysis

Second order analysis consists of ranking a pair of control options to a pair of ranks. For example, someone can choose first and second control options on third and fourth or

<sup>5</sup> The first order effect measures the average attraction that a treatment option has when it is coupled with another treatment option.

<sup>6</sup> The second order effect detects the positive (or negative) power of combination of two coupled treatment options.

fourth and third ranking positions. The rank can be in ordered or unordered positions. Therefore, there are two types of second order functions. Again following Mallow’s approach, let

$$\pi \rightarrow \delta_{\{i,i'\}\{\pi(j)\pi(j')\}} \begin{cases} =1 & \text{if } \{\pi(j)\pi(j')\} = \{i, i'\} \\ =0 & \text{otherwise} \end{cases}$$

then, the general, unordered second ordered ( $V_2$ ) function will be of the following form

$$\sum_{i,i',j,j'} a_{ii',jj'} \delta_{\{ii'\}\{\pi(j)\pi(j')\}}$$

where,  $a_{ii',jj'}$  are chosen so that  $V_2$  is orthogonal to  $V_0 \oplus V_1$ . In this case, the order does not matter and it has two two-dimensional subspaces so the second ordered unordered effect has a four-dimensional space. In a similar way we can find the higher order function, which is beyond the objective of this paper.

Appendix A2: Computation of first order analysis

First order space decomposed in two invariant subspaces for each preference. For example, first order, first preference space  $V^{(3,1)} (=V_1)$  with its data vector  $f^{(3,1)}$  consists of two invariant subspaces:  $V_0^{(3,1)}$  mean effect with its data vector  $f_0^{(3,1)}$  and  $V_1^{(3,1)}$  the first order pure effects with its data vector  $f_1^{(3,1)}$ .  $f_0^{(3,1)}$  is found by projecting  $f^{(3,1)}$  onto  $V_0^{(3,1)}$  and  $f_1^{(3,1)}$  is found by projecting  $f^{(3,1)}$  onto  $V_1^{(3,1)}$ . And finally, this gives the following decomposition:

$$f^{(3,1)} = \begin{pmatrix} 142 \\ 400 \\ 93 \\ 112 \end{pmatrix}, \quad f_0^{(3,1)} = \begin{pmatrix} \frac{747}{4} \\ \frac{747}{4} \\ \frac{747}{4} \\ \frac{747}{4} \end{pmatrix}, \quad f_1^{(3,1)} = \begin{pmatrix} -\frac{179}{4} \\ \frac{853}{4} \\ -\frac{375}{4} \\ -\frac{299}{4} \end{pmatrix}$$

$$f^{(3,1)} = f_0^{(3,1)} + f_1^{(3,1)}$$

Appendix A3: Computation of second order analysis

This is second order unordered effects. The second order unordered effect space  $V^{(2,2)} (=V_2)$  with its data vector  $f^{(2,2)}$  decomposes into three invariant subspaces:  $V_0^{(2,2)}$  mean effect with its data vector  $f_0^{(2,2)}$ ,  $V_1^{(2,2)}$  first order effect with its data vector  $f_1^{(2,2)}$ , and two dimensional second order unordered pure effect  $V_2^{(2,2)}$  with its data vector  $f_2^{(2,2)}$ . In particular, the decomposition for the pair of liquid and bait treatment options is illustrated below. The data vector  $f^{(2,2)}$ , the number homeowners

who favor control options as two most preferred options, uniquely can be written as sum of  $f_0^{(2,2)}$ ,  $f_1^{(2,2)}$ ,  $f_2^{(2,2)}$ , and they are orthogonal to each other.

$$\begin{aligned} \|f_0^{(2,2)}\|^2 &= \|f_0^{(2,2)}\|^2 + \|f_1^{(2,2)}\|^2 + \|f_2^{(2,2)}\|^2 \\ 188301 & \quad 93001.5 \quad 66678.5 \quad 28621 \\ \begin{pmatrix} 180 \\ 12 \\ 10 \\ 370 \\ 46 \\ 129 \end{pmatrix} &= \begin{pmatrix} 124.5 \\ 124.5 \\ 124.5 \\ 124.5 \\ 124.5 \\ 124.5 \end{pmatrix} + \begin{pmatrix} 25.5 \\ -17 \\ -180 \\ 180 \\ 17 \\ 25.5 \end{pmatrix} + \begin{pmatrix} 30 \\ -95.5 \\ 65.5 \\ 65.5 \\ -95.5 \\ 30 \end{pmatrix} \\ f^{(2,2)} &= f_0^{(2,2)} + f_1^{(2,2)} + f_2^{(2,2)} \end{aligned}$$

Appendix A4: An illustration of Mallows’ approach

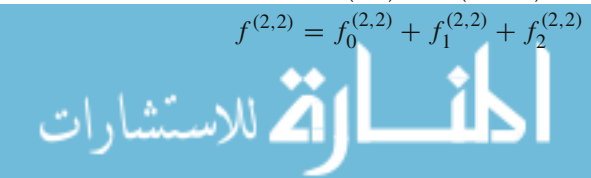
For income category 0, we calculate the inner product between the function  $f_1^{(2,2)}$  and a function  $f_T^{2,2} \in V_T^{(2,2)}$  where  $T$  represents the treatment option. This function  $f_T^{(2,2)}$  identifies the elements of  $f_1^{(2,2)}$  “containing” treatment option (1, 2, 3, 4) with 1 and those “non containing”  $T$  with 0, if  $T = 1$   $f_{T=1}^{2,2} = (1, 1, 1, 0, 0, 0)$ . The contribution of the attribute treatment 1 is  $f_{1,T=1}^{2,2} = f_1^{(2,2)} \cdot f_{T=1}^{2,2}$ .

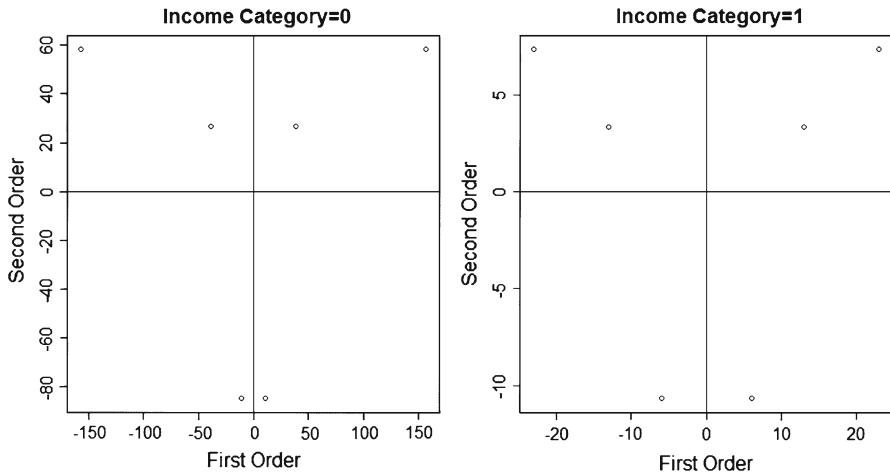
Income less than \$150K (income category = 0)

$$\begin{aligned} f^{(2,2)} &= \begin{pmatrix} 173 \\ 12 \\ 9 \\ 323 \\ 34 \\ 96 \end{pmatrix} = \begin{pmatrix} 107.83 \\ 107.83 \\ 107.83 \\ 107.83 \\ 107.83 \\ 107.83 \end{pmatrix} + \begin{pmatrix} 38.50 \\ -11.00 \\ -157.00 \\ 157.00 \\ 11.00 \\ -38.50 \end{pmatrix} + \begin{pmatrix} 26.67 \\ -84.83 \\ 58.17 \\ 58.17 \\ -84.83 \\ 26.67 \end{pmatrix} \begin{matrix} 1, 2 \\ 1, 3 \\ 1, 4 \\ 2, 3 \\ 2, 4 \\ 3, 4 \end{matrix} \\ f^{(2,2)} &= f_0^{(2,2)} + f_1^{(2,2)} + f_2^{(2,2)} \end{aligned}$$

Income equal or more than \$150K (income category = 1)

$$\begin{aligned} f^{2,2} &= \begin{pmatrix} 7 \\ 0 \\ 1 \\ 47 \\ 12 \\ 33 \end{pmatrix} = \begin{pmatrix} 16.67 \\ 16.67 \\ 16.67 \\ 16.67 \\ 16.67 \\ 16.67 \end{pmatrix} + \begin{pmatrix} -13.00 \\ -6.00 \\ -23.00 \\ 23.00 \\ 6.00 \\ 13.00 \end{pmatrix} + \begin{pmatrix} 3.33 \\ -10.67 \\ 7.33 \\ 7.33 \\ -10.67 \\ 3.33 \end{pmatrix} \begin{matrix} 1, 2 \\ 1, 3 \\ 1, 4 \\ 2, 3 \\ 2, 4 \\ 3, 4 \end{matrix} \\ f^{(2,2)} &= f_0^{(2,2)} + f_1^{(2,2)} + f_2^{(2,2)} \end{aligned}$$





**Fig. A1** Joint first and second order effects by income

To get the first value in Table 5 with income category = 0, we need to follow this calculation:

$$f_{1,T=1}^{2,2} = f_1^{(2,2)} \cdot f_{T=1}^{2,2} = \begin{pmatrix} 38.50 \\ -11.00 \\ -157.00 \\ 157.00 \\ 11.00 \\ -38.50 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = -129.5$$

We can plot the frequencies of choices by first and second order effects to better understand data which is shown in Fig. A1.

## Appendix A5

### Sample ranking question

There are different alternatives that homeowners can choose to protect their homes from Formosan Subterranean Termites. We did like you to evaluate and rank your preferences from the alternatives listed below. Please indicate your ranking selection on the “Rank” space.

- 1 = First preference
- 2 = Second preference
- 3 = Third preference
- 4 = Fourth preference

## Rank

- *Alternative 1* Do not engage in any sort of activities, such as contracting with a pest control operator or company, to protect against termites. This option will cost you no money. With no form of termite protection or control, however, the chance that your home will be attacked by termites over the next 5 years is significant.
- *Alternative 2* Contract with a pest control operator or company to install a liquid termite prevention solution (an insecticide that is applied in a trench dug around your home) around the exterior of your house. The cost of this option is as follows (based on a hypothetical 2,000 square foot home): initial inspection and installation fee = \$750, annual renewal fees = \$113 per year (including first year). This equates to an average cost over the next 5 years of \$0.13 (13 cents)/square foot per year. With this service you will receive one home inspection per year. The contract lasts for 5 years.
- *Alternative 3* Contract with a pest control operator or company to install a termite baiting system around the exterior of your home (small, self-contained insecticide bait stations are placed into the ground around the perimeter of your house) to assist in preventing termite infestation. The cost of this option is as follows (based on a hypothetical 2,000 square foot home): initial inspection and installation fee = \$2,000, annual renewal fees = \$450 per year (including the first year). This equates to an average cost over the next 5 years of \$0.43/square foot per year. With this service you will receive a minimum of one inspection per month. The contract lasts for 5 years.
- *Alternative 4* Contract with a pest control operator or company to install a liquid termite prevention solution around the exterior of your house PLUS a termite bait system which further prevents termites. The cost of this option is as follows (based on a hypothetical 2,000 square foot home): initial inspection and installation fee = \$2,750, annual renewal fees = \$563 per year (including the first year). This equates to an average cost over the next 5 years of \$0.56 (56 cents)/square foot per year. With this service you will receive a minimum of one inspection per month. The contract lasts for 5 years.

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